re(Solve) MATHS BY INQUIRY Forensic Investigations

Y10

Summary of learning goals

- Students examine real-world case studies of probability being applied in misleading ways. They assess and explain the flaws in the mathematical methodology used.

Australian Curriculum: Mathematics (Year 10)

ACMSP246: Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence.

ACMSP247: Use the language of 'if...then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language.

ACMSP253: Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data.

Summary of lessons

Who is this sequence for?

- This sequence is for students who are familiar with a suite of tools for working with probability, especially students who have an understanding of independent and dependent events and are able to identify mistakes in language used to communicate ideas around probability.

Lesson 1: Forensic Investigations

Students examine real-world case studies of probability being applied in misleading ways. They assess and explain the flaws in the mathematical methodology used.

reSolve: Mathematics by Inquiry is funded by the Australian Government and is managed by the Australian Academy of Science.

Reflection on this sequence

Rationale

In this sequence students investigate real-world applications of probability. Students are encouraged to turn a critical eye to the use of mathematics in the media to sway opinion and to test their own understanding of independent and dependent events. There is also an emphasis on accurate and inaccurate ways of communicating mathematical concepts to the general public.

reSolve mathematics is purposeful

- Students explore real-world examples of probability in the courtroom and investigate how mathematics swayed popular opinion on each case.
- The task supports a rich interpretation of the Australian Curriculum: Mathematics through the manipulation, representation and interpretation of probability to tell a story.

reSolve tasks are inclusive and challenging

- Students in groups are assigned one of three different case studies of varying complexity, allowing the teacher to determine the most appropriate case study for each group.

reSolve classrooms have a knowledge-building culture

- Students investigate their case studies in groups, discussing and collaborating throughout their exploration.
- Students present findings to their class, illustrating the flaws in their case by presenting their own comparative examples.

Acknowledgements

Colmez S & Schneps L, 2013, *Math on trial: how numbers get used and abused in the courtroom*. Basic Books: New York

LESSON 1

re(Solve) MATHS BY INQUIRY

Forensic Investigations

Forensic Investigations

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About this lesson

Students examine real-world case studies of probability being applied in misleading ways. They assess and explain the flaws in the mathematical methodology used.

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Mathematical purpose

- Students will determine appropriate uses of probability in real-world contexts.

Learning intention

- To analyse and critique real-world applications of probability.

Time

A lesson of approximately 1 hour.

Resources

- reSolve PDF *1a Case Study Sheets*

Vocabulary

- dependent/independent event
- Simpson's paradox

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Teacher background information

This task adapts three real-world cases, as outlined in *Math on trial: how numbers get used and abused in the courtroom* by Leila Schneps and Coralie Colmez, to demonstrate examples of misleading mathematics being presented in court. reSolve PDF *1a Case Study Sheets* draws from *Math on trial* and original courtroom documents to present all the necessary information to students. Given below is a summarised version of each case and its eventual outcome for teachers' reference.

Case 1: Sally Clark

Summary

In 1998 Sally Clark was accused of murdering her two sons Christopher (died in 1996, 10 weeks old) and Harry (died in 1998, 8 weeks old). During the trial Sir Roy Meadow, a renowned paediatrician, argued that the chance of one baby dying of SIDS in the Clark family was 1 in 8543. He squared this value to arrive at a 1 in 73 million chance that two babies in the same family could die of SIDS.

Verdict

Sally Clark was found guilty of both murders.

More information

The 1 in 73 million statistic was widely criticised at the time, for several reasons:

- It treats two SIDS deaths in the family as independent events, but studies suggest that after a single SIDS death a second one becomes much more likely due to unknown genetic/environmental factors.
- The 1 in 8543 possibility does not take into consideration that the family had a high risk factor for SIDS.
- The 'prosecutor's fallacy'—even if the 1 in 73 million statistic were accurate, it does not equal the chance that Sally was innocent. Statistically, double infant murder is less likely than a double SIDS death.

Sally Clark's conviction was overturned in 2003.

The Sally Clark case includes opportunities for students to do independent online research and to engage in interesting social issues.

Case 2: Janet Collins

Summary

Janet and Malcolm Collins were accused of mugging Mrs Juanita Brooks because they matched the descriptions of the thieves. The evidence against the Collins was considered inconclusive and the prosecution attempted to prove their guilt by demonstrating how unlikely it was that another couple would match the descriptions given.

Verdict

Janet and Malcom were both found guilty of second-degree robbery.

More information

Upon appeal, it was observed that:

- The probabilities used in the court case relied on loose estimates rather than statistical research.
- The product rule applied only to independent events and many of the given events were not independent.
- The cited 1 in 12 million chance does not mean that the Collins were the guilty couple—in fact, given one couple that met the description, there was a 40% chance that another couple in the area would also meet the description.

The Collins' judgment was overturned in 1968.

The Janet and Malcolm Collins case is very accessible. Students should enjoy designing their own similar examples.

Case 3: UC Berkeley

Summary

In 1973 the University of California in Berkeley accepted 44% of male applicants and 35% of female applicants to its graduate school. Concerned that it could be faced with an accusation of sex bias, the university investigated further. It discovered that of its six departments, four accepted a higher proportion of female applicants, and the other two had only a very slightly higher percentage of males.

Verdict

There was no systematic sex bias from the university. It was found that women tended to apply to competitive departments with low admission rates, whereas men applied to less competitive departments with high admission rates.

More information

This is an example of Simpson's paradox, whereby one trend appears in several distinct groups of data, but when the data are combined the trend reverses or disappears. This case is intriguing and counterintuitive. Students studying this example should be encouraged to research and design their own example of the paradox.

Introduction

Observe that mathematics can be very influential in the courtroom. Discuss some reasons why mathematical reasoning might be so persuasive for a jury or judge.

Explain to students that they will be exploring real-world examples of statistics and probability used in court cases. Do not give away at this stage that the first two examples were determined to be miscarriages of justice and that the third was concluded to be non-noteworthy (i.e. that these are all examples of misleading or misused mathematical reasoning).

Resources: Divide students into groups of three or four and give each group one of the three study sheets from reSolve PDF *1a Case Study Sheets*.

Exercise your own judgement to divide students and assign case studies—the three different case studies all require different levels of engagement and will appeal to different interests (see Teacher background information).

Exploration

Students should read through their case study sheet, discuss the mathematics used and present their conclusions to the class. A possible process could be:

1. Understand and clarify: in their groups, students read through the case study sheet and discuss the key elements of the story.
2. Students identify the mathematics used and test the hypothesis of the story to show that the claims made are based on a plausible analysis of the data. For example:
   a. Clark case: calculating how the 1 in 73 million statistic and the 'once every hundred years' comment might have been reached.
   b. Collins case: calculating how the 1 in 12 million statistic was found.
   c. UC Berkeley case: ensuring that all the numbers are accurate and finding the percentage of each gender admitted into each department.
3. Students ask critical questions about the mathematical strategy used. Ask them to consider what strategy was used and whether they can identify any problems with it. For example:
   a. Clark case: The two probabilities are not independent and should not be multiplied (might require some research into SIDS).
   b. Collins case: The probabilities were estimates and probably different in reality, and some of the listed probabilities were not independent. (Prompt: *Are the chances of someone having a beard and having a moustache independent?*).
   c. UC Berkeley case: This might require some research into Simpson's paradox.
4. Students present their findings to the class to convince others that the mathematical strategy used was misleading.

Reflection

Students apply the mathematical strategy used in the case study to a new context, to illustrate any problems with the reasoning used. For example:

1. Clark case: Devise a story in which the same event happens twice in ways that seem unrelated, but are actually not independent.
2. Collins case: Create an 'estimated' list of probabilities that describes a student in the class and use this list to prove the student is 'one in a million'.
3. UC Berkeley case: Research more examples of Simpson's paradox and devise a variant. For example, the classic 'someone moving from New Zealand to Australia raises the average IQ of both countries' joke, or cricket or basketball averages.

Further activities

Prosecutor's fallacy

Look into the concept of the 'prosecutor's fallacy'. How did this play into the Collins and Clark cases?